



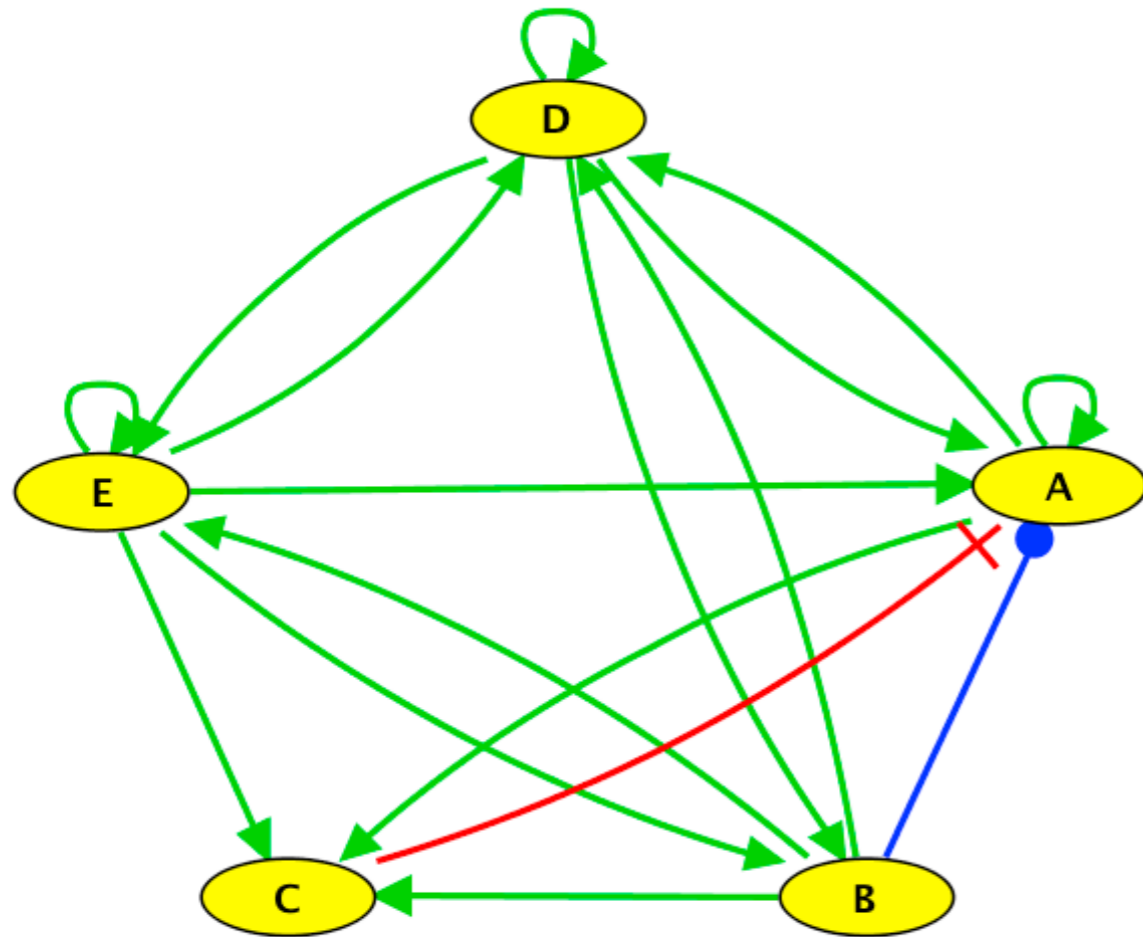
# Computational analysis of the dynamics of logical regulatory graphs

## Hierarchical compaction of state transition graphs

Duncan Berenguier

Claudine Chaouiya, Elisabeth Remy, Denis Thieffry

# Model — Logical Regulatory Graph



$$A = (B \ \& \ E \ \& \ !C) \ | \ (A \ \& \ !B \ \& \ D \ \& \ E)$$

$$B = D \ \& \ E$$

$$C = A \ \& \ B \ \& \ E$$

$$D = D \ | \ (A \ \& \ (B \ | \ E))$$

$$E = (D \ \& \ B) \ | \ (E \ \& \ D \ \& \ !B)$$

A **node** is a **regulatory component** associated with a **discrete value** denoting its **level of activity**, e.g.

0 : Inactive

1 : Active

An **arc** is a **regulatory interaction**, whose effect (**activation**, **inhibition**, **dual**) is described by the logical function of its target

The **logical function** describe how the level of activity of a component evolve depending on its regulators

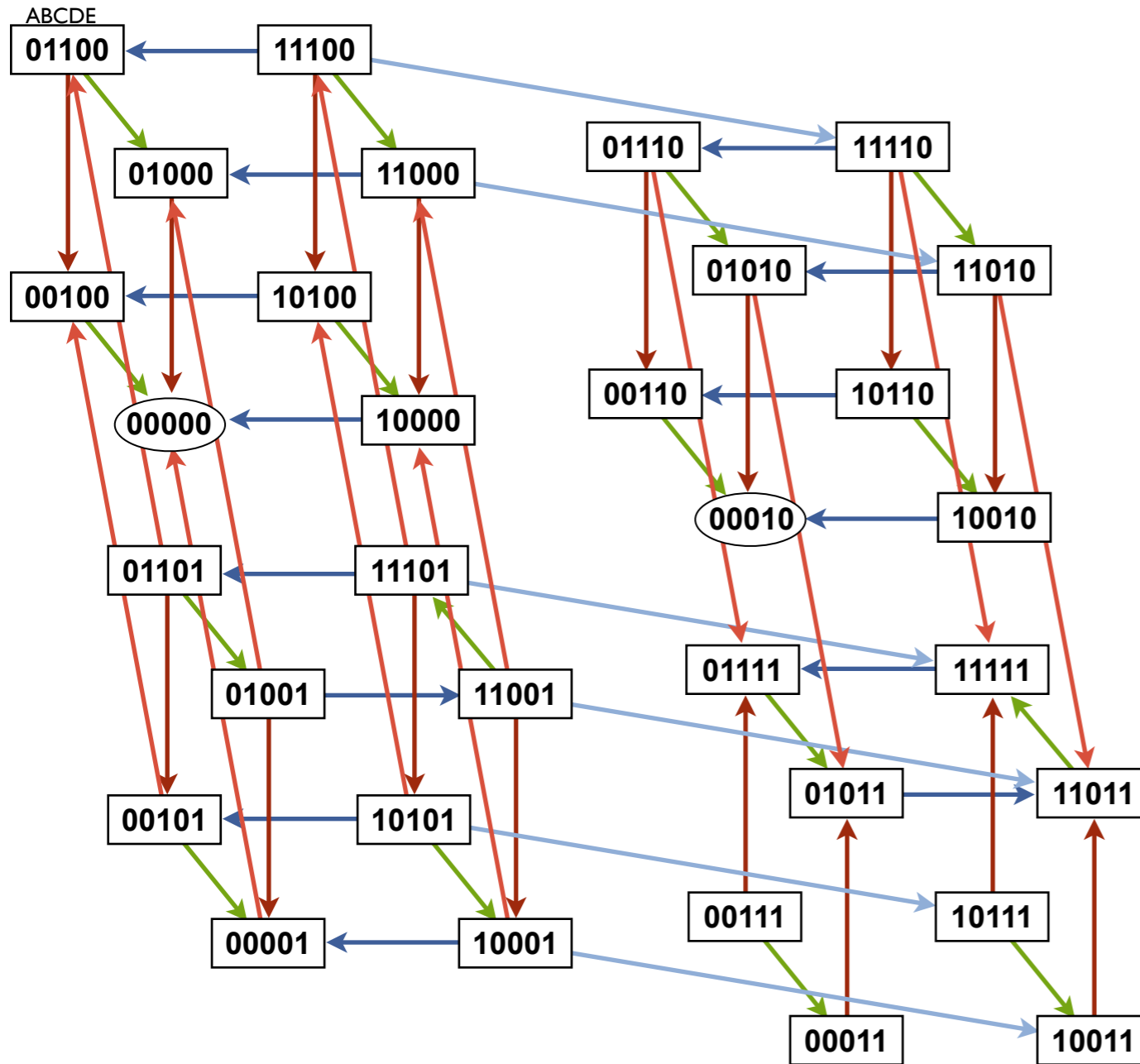
We consider several **updating strategies** :

synchronous

**asynchronous**

priorities classes

# Dynamics – State Transition Graph



A node represents a **state** of the system

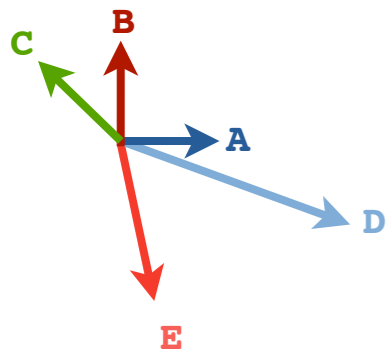
A **transition** between two states indicates a possible evolution

Problem : **Combinatory explosion**

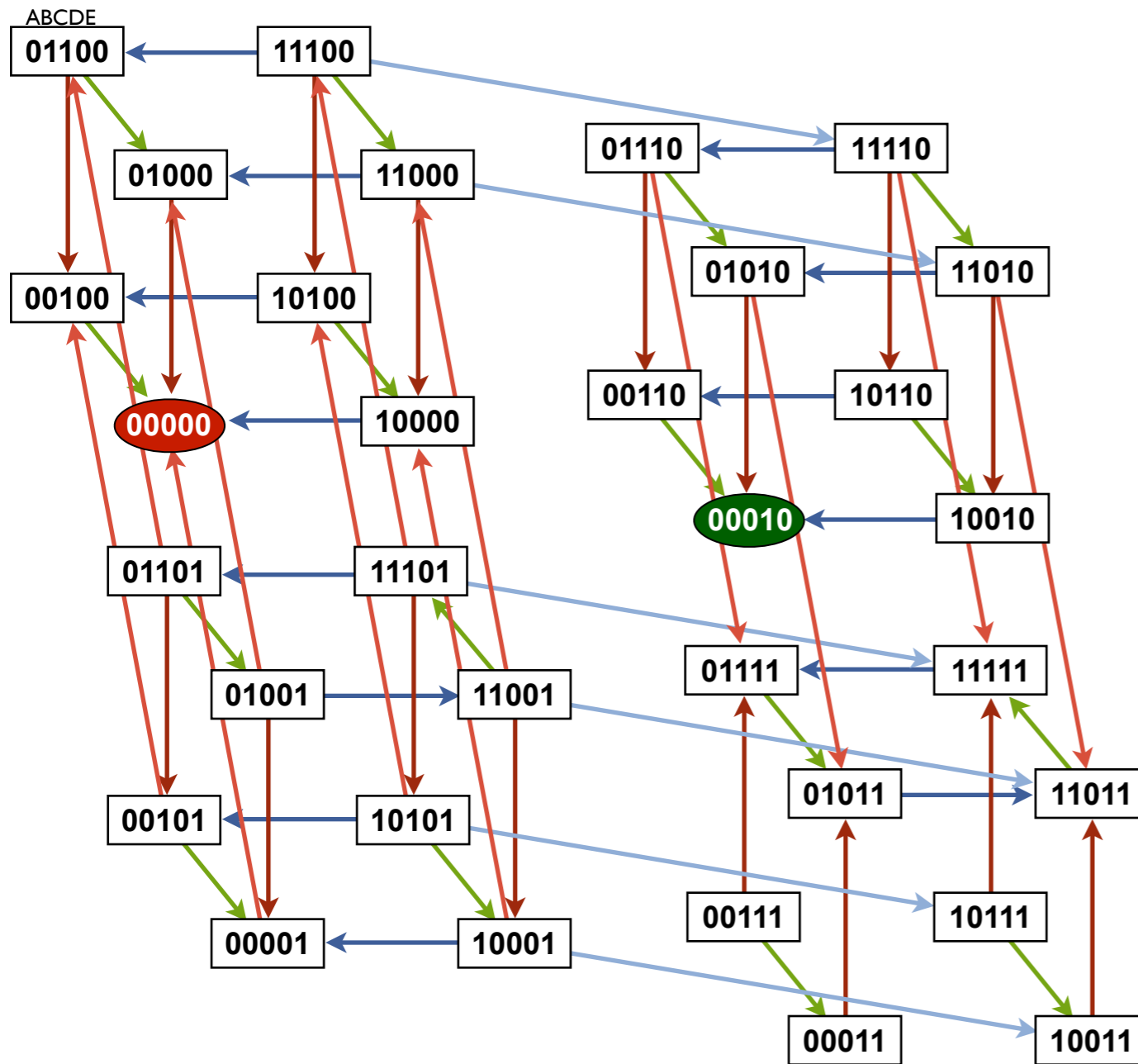
5 components  $\Rightarrow$  32 states

20 components  $\Rightarrow$  over 1 million states

N components  $\Rightarrow 2^N$  states

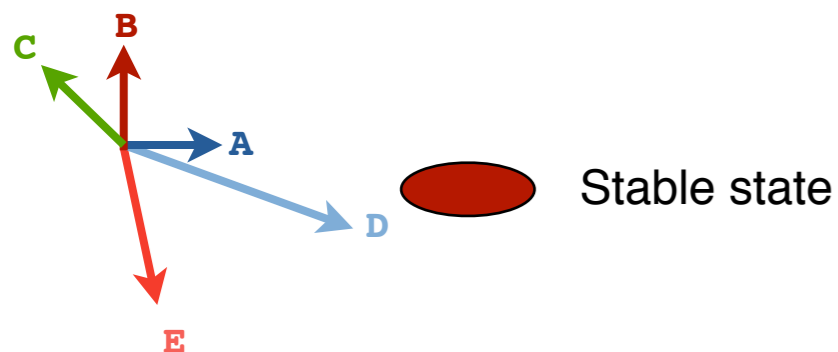


# Dynamics – State Transition Graph

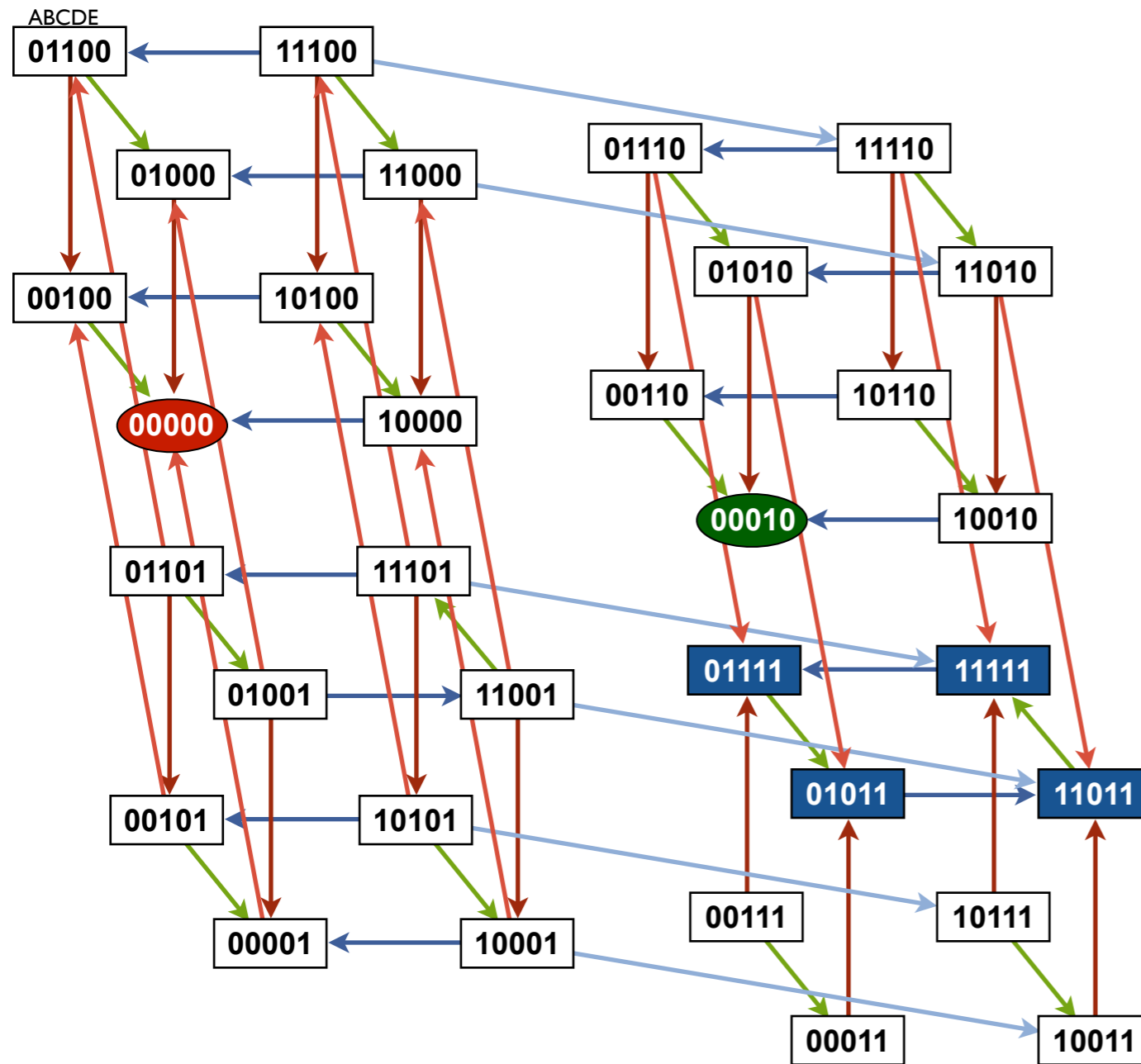


The asymptotic behaviour is described by **attractors**

Attractors correspond to **terminal strongly connected components**



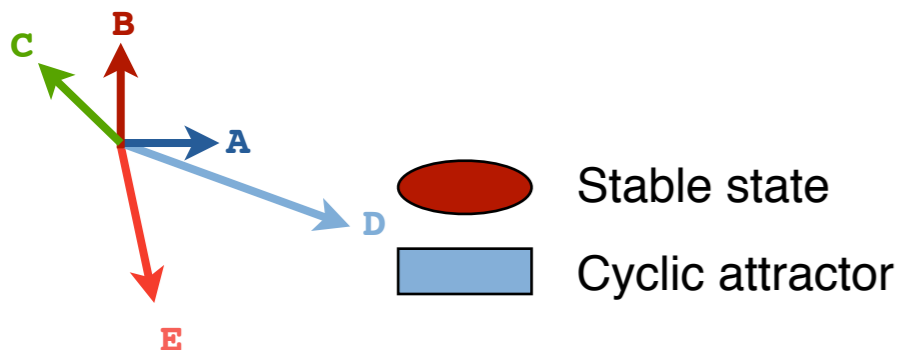
# Dynamics – State Transition Graph



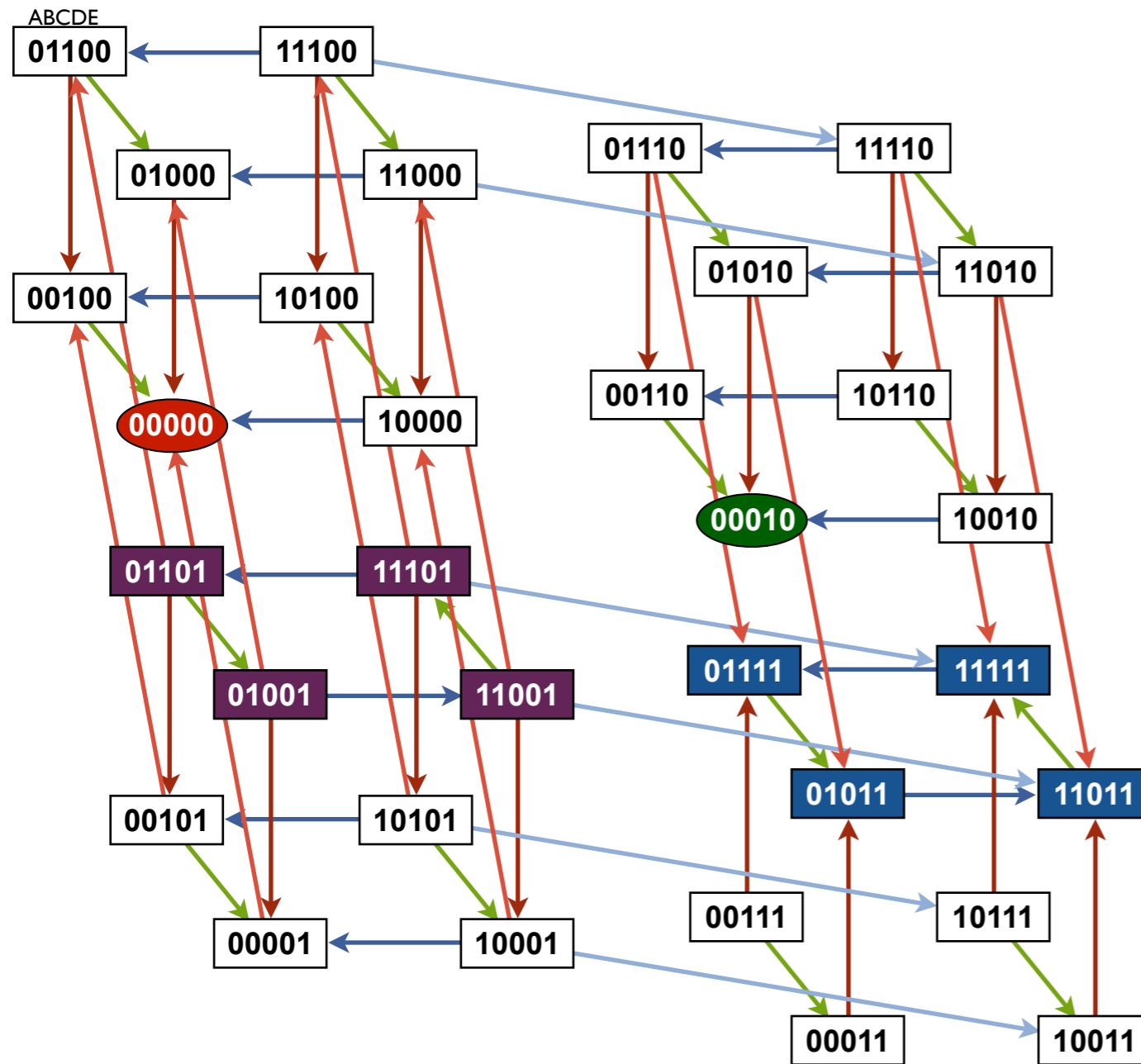
The asymptotic behaviour is described by **attractors**

Attractors correspond to **terminal strongly connected components**

The oscillatory behaviours are described by **cycles**



# Dynamics – State Transition Graph

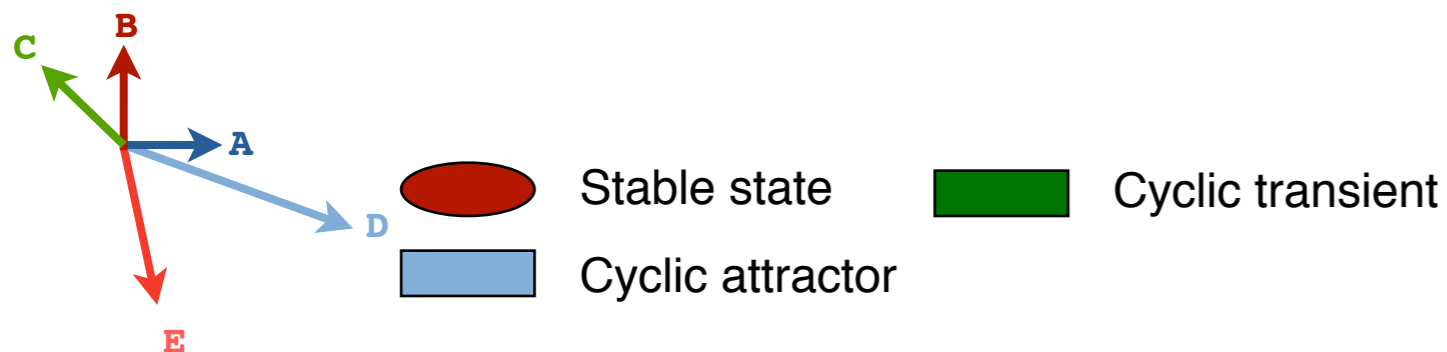


The asymptotic behaviour is described by **attractors**

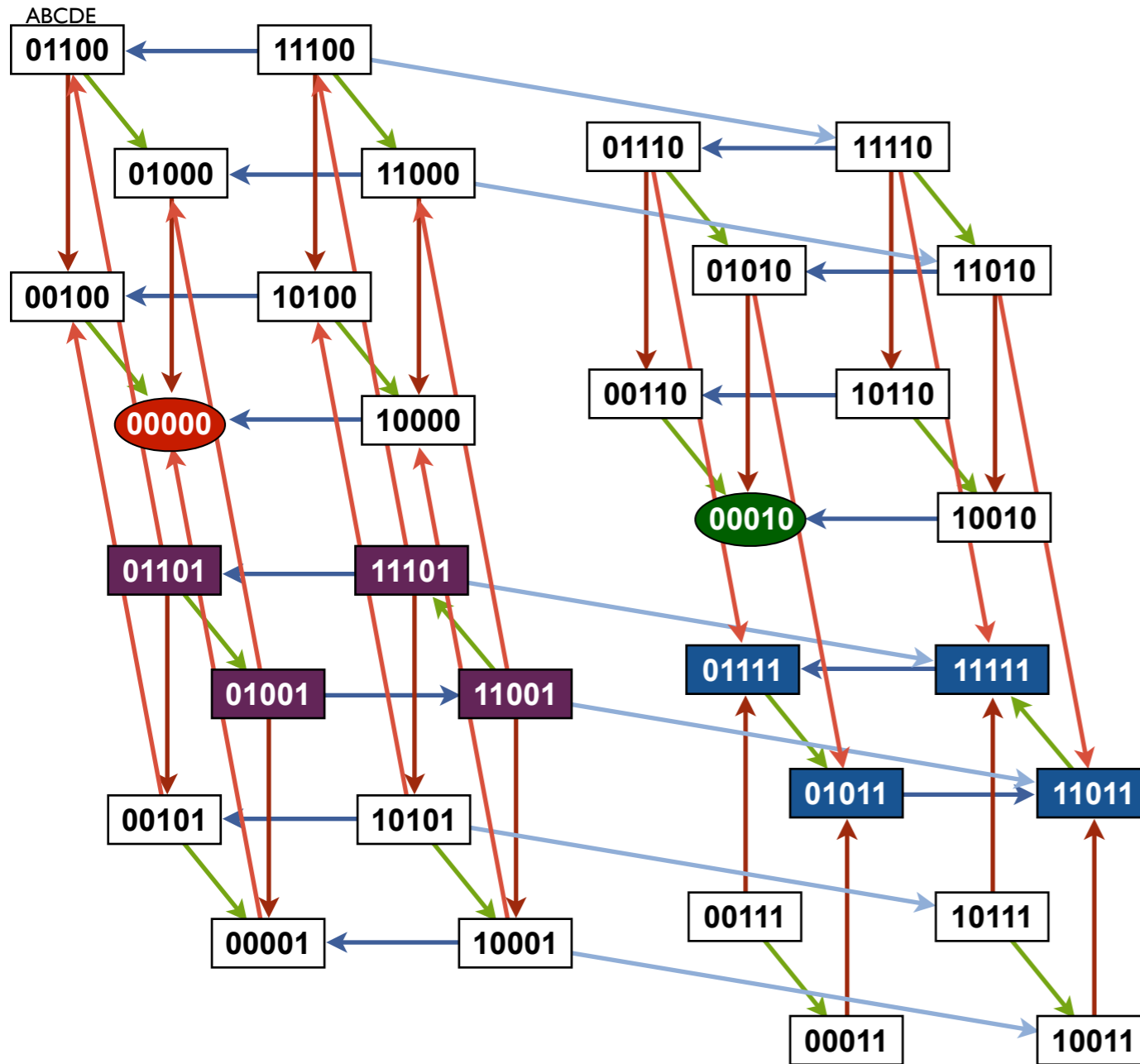
Attractors correspond to **terminal strongly connected components**

The oscillatory behaviours are described by **cycles**

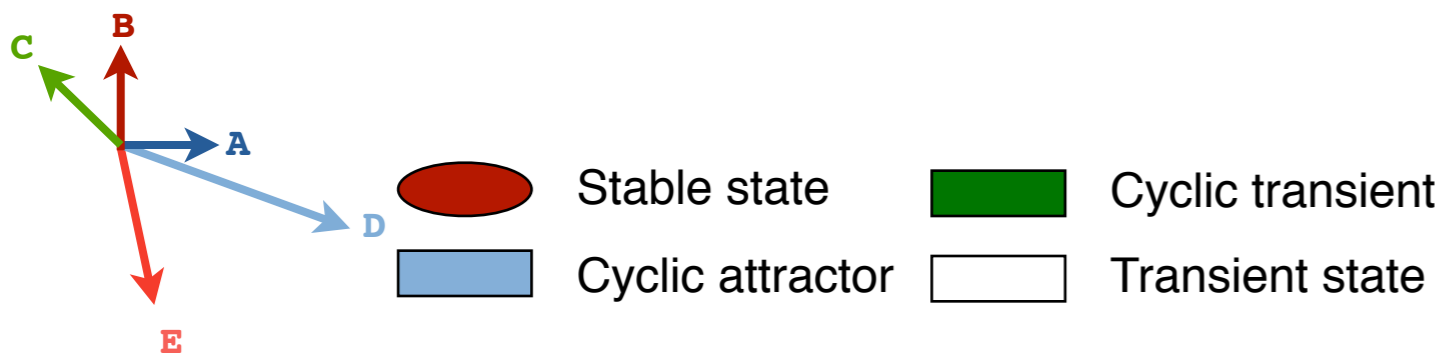
Transient oscillatory behaviours correspond to **transient** strongly connected components



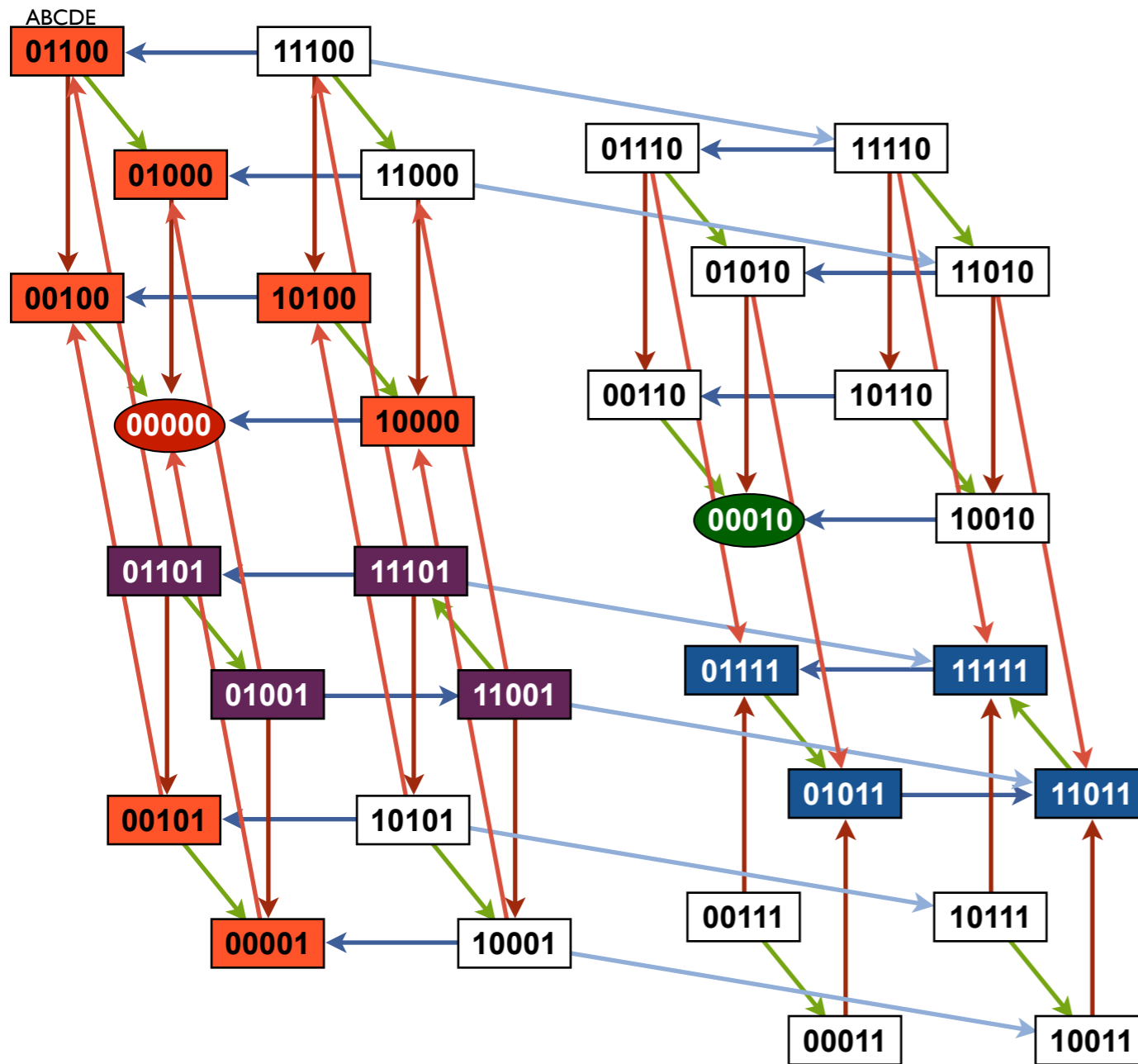
# Dynamics – State Transition Graph



What to do with the remaining transient states ?

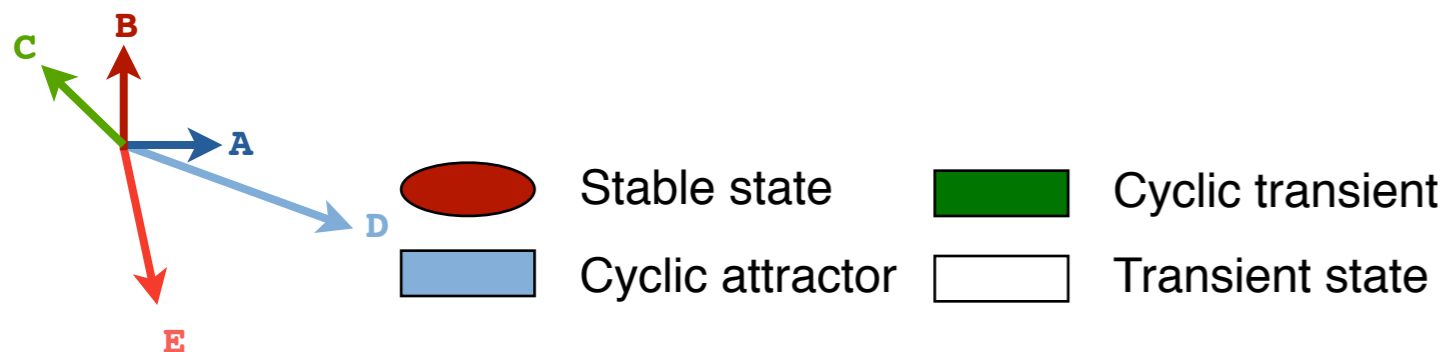


# Dynamics – State Transition Graph



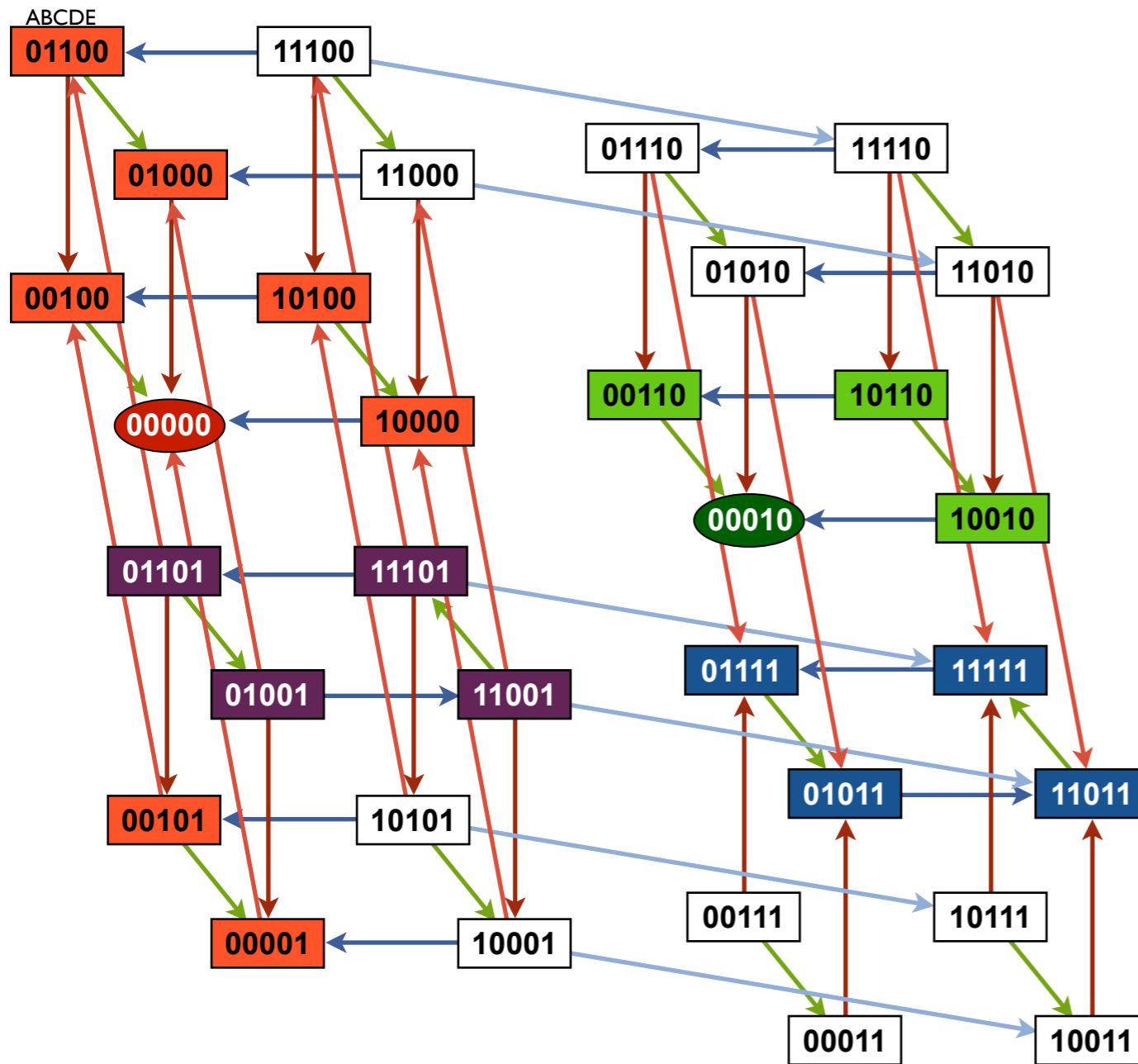
An (extended) **basin of attraction** is the set of nodes reaching a (set of) attractor(s)

We **classify** the transient states depending on their appartenance to a basin of attraction



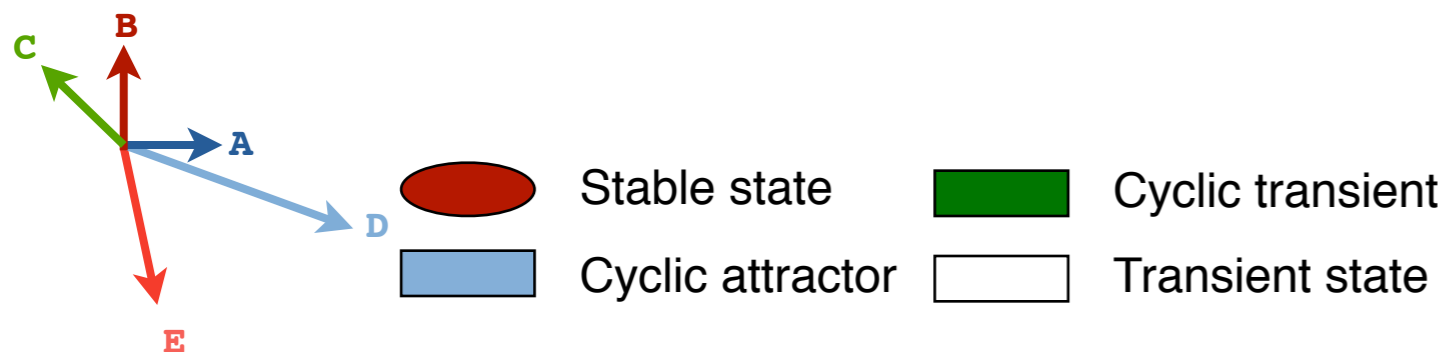


# Dynamics – State Transition Graph

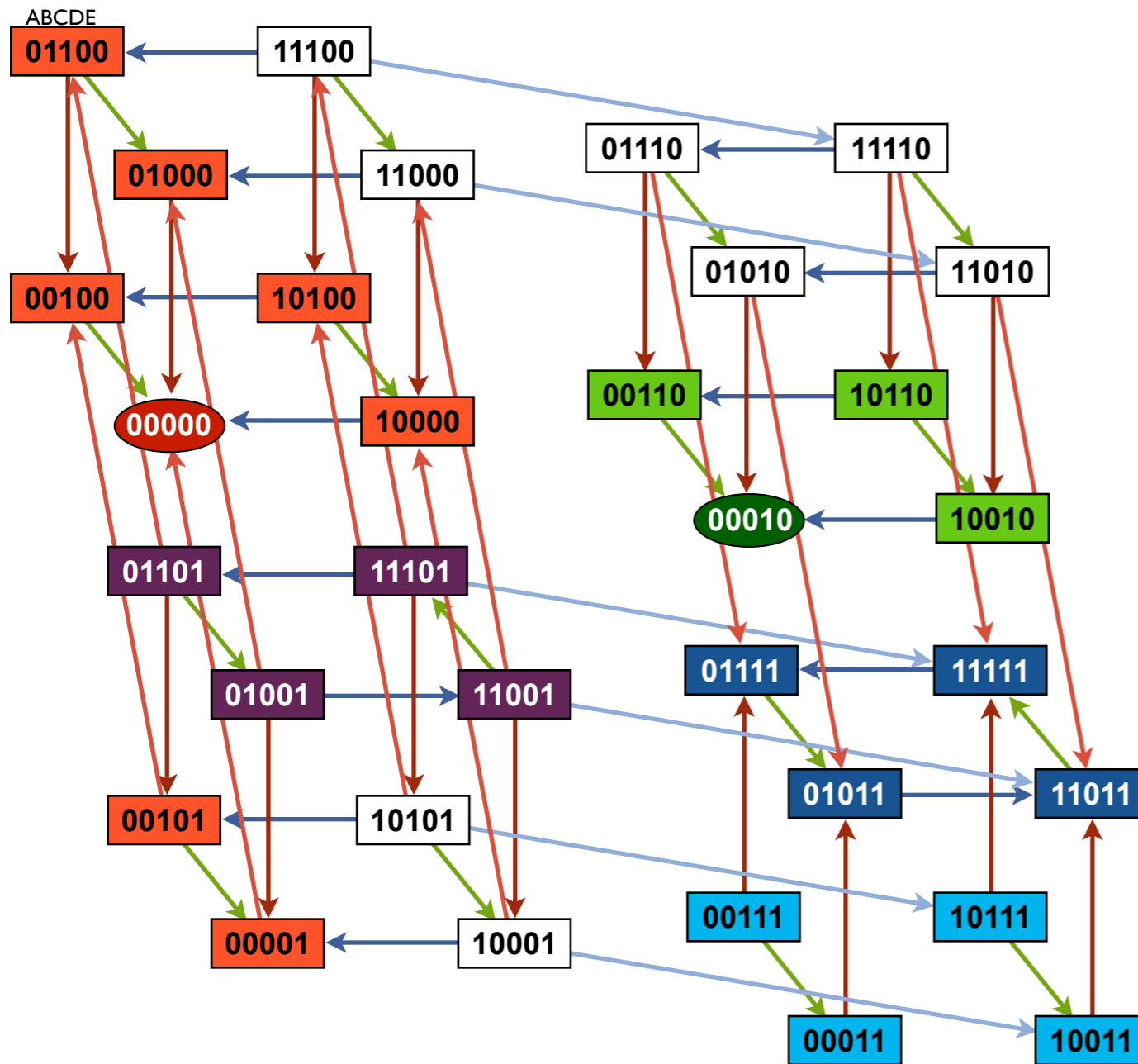


An (extended) **basin of attraction** is the set of nodes reaching a (set of) attractor(s)

We **classify** the transient states depending on their appartenance to a basin of attraction

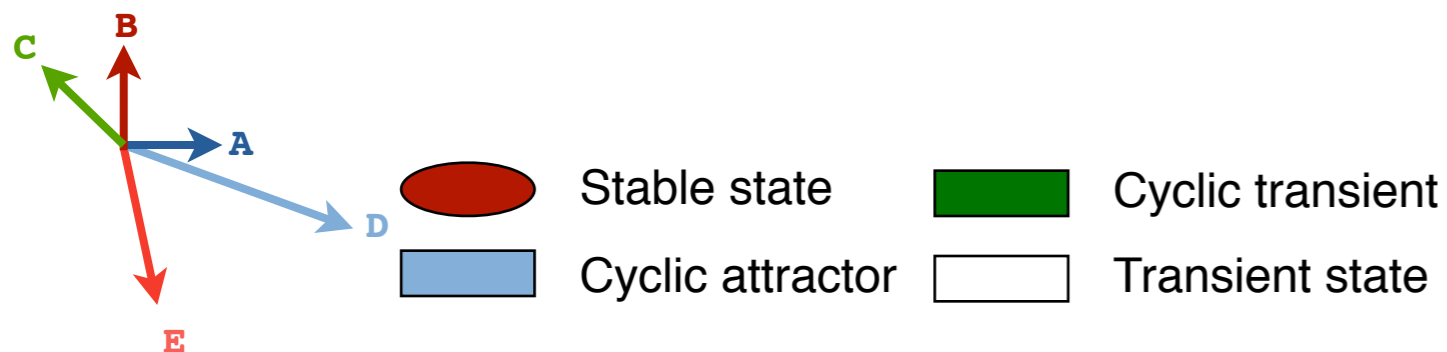


# Dynamics – State Transition Graph

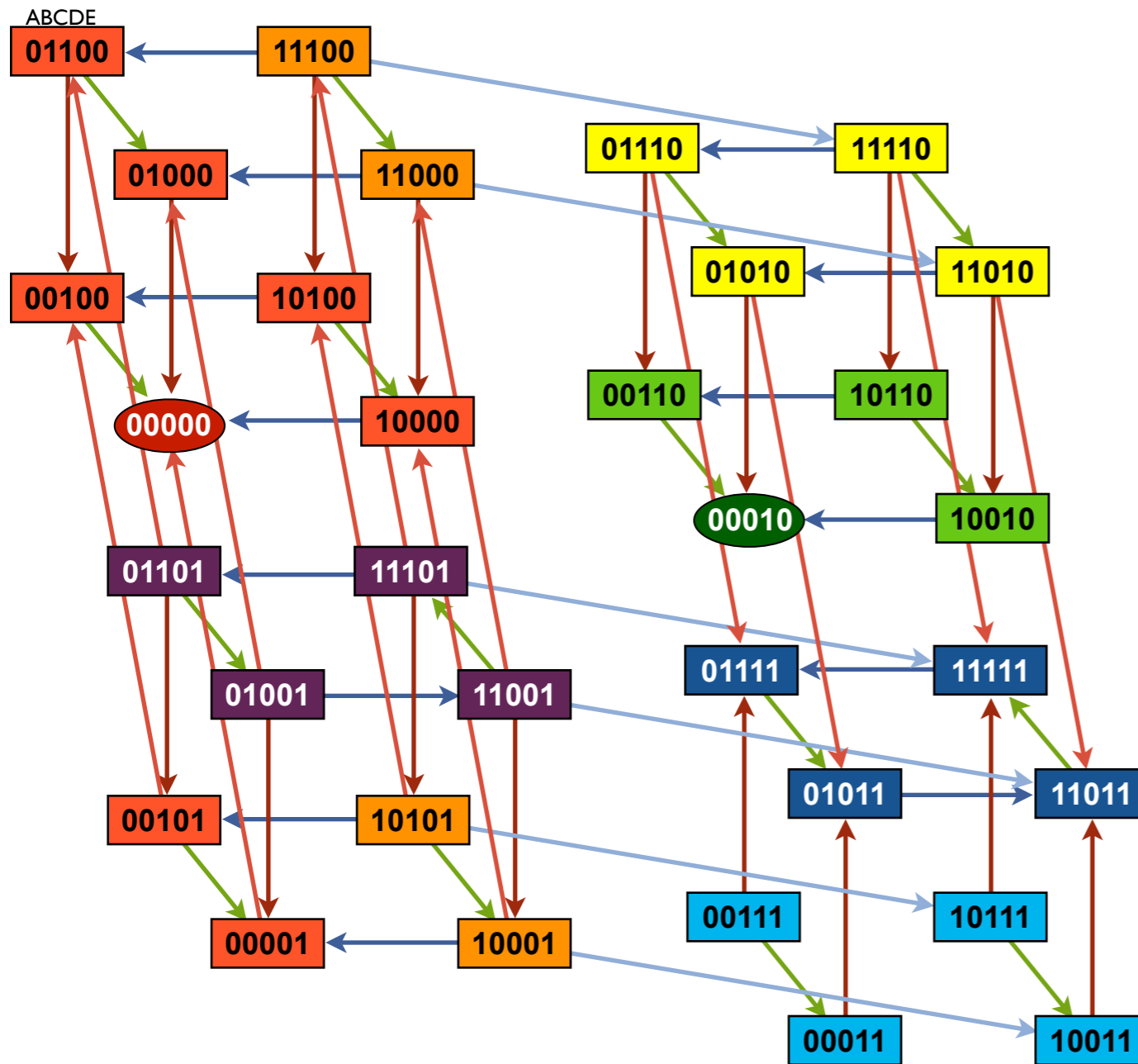


An (extended) **basin of attraction** is the set of nodes reaching a (set of) attractor(s)

We **classify** the transient states depending on their appartenance to a basin of attraction

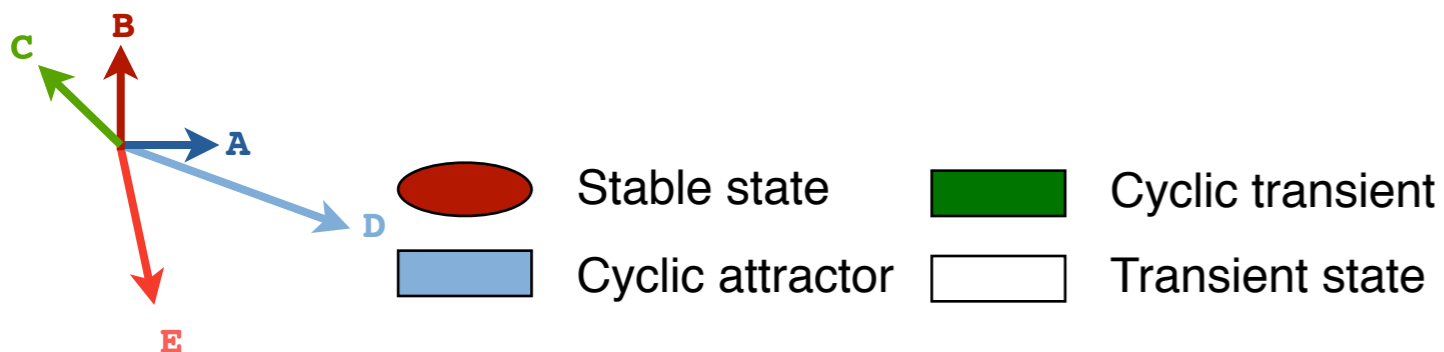


# Dynamics – State Transition Graph

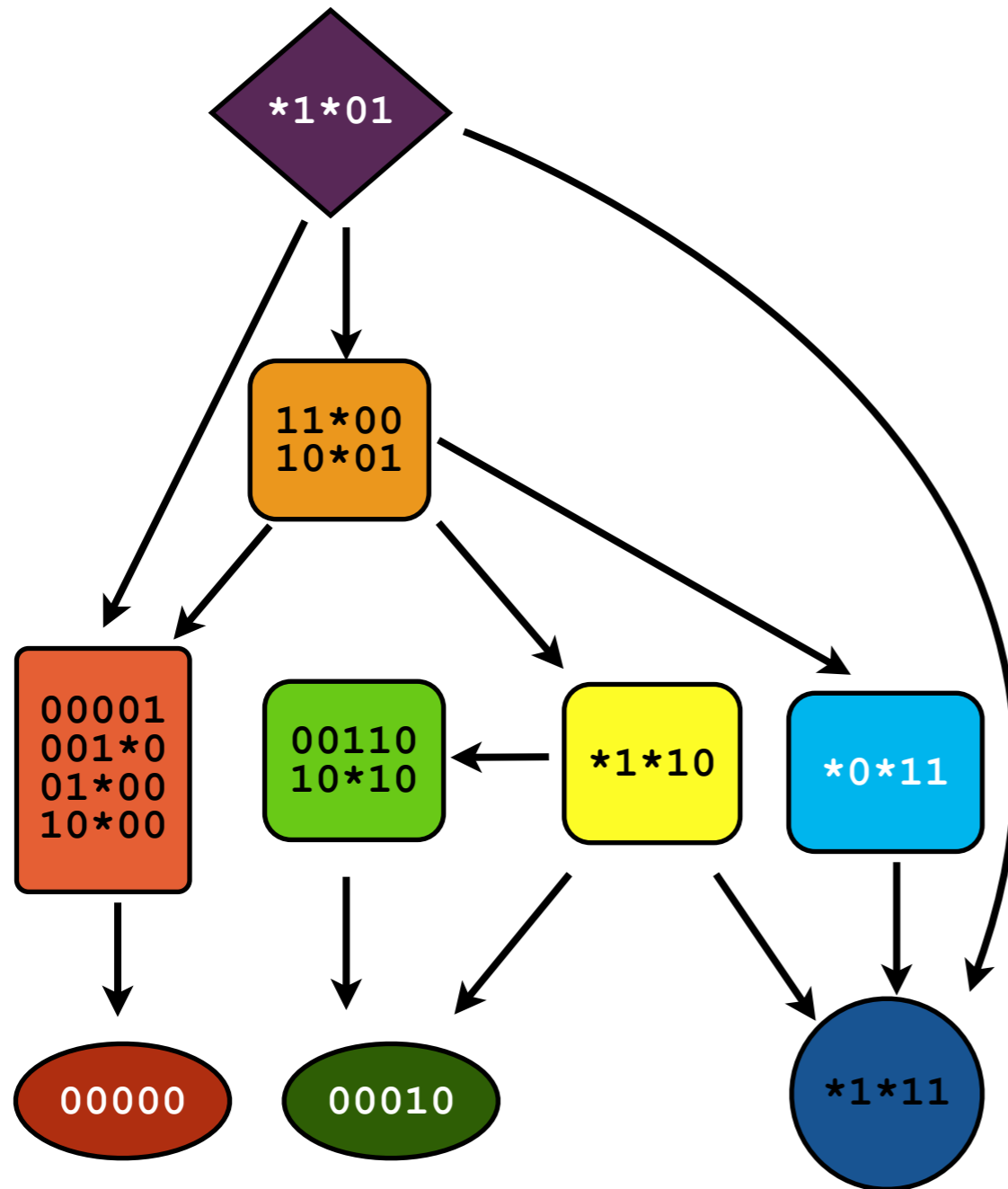


An (extended) **basin of attraction** is the set of nodes reaching a (set of) attractor(s)

We **classify** the transient states depending on their appartenance to a basin of attraction



# Dynamics – Hierarchical Transition Graph



○ Stable state

○ Cyclic attractor

◇ Cyclic transient

□ Irreversible component

This classification of the states allows us to generate a **hierarchical transition graph**

A **node** represents a **set of states** (represented by schemata)

The graph is **smaller** and easier to analyse than the state transition graph

## Take home message

The proposed compacted **hierarchical representation** of state transition graphs **eases the analysis** of the dynamics of large logical models

## Prospects

What is the impact of **model reductions** on the organisation of hierarchical transition graphs ?

More generally, can we define a **class of models** producing the same hierarchical organisation ?

# Thank you for your attention !



**Belgian Inter-university  
Attraction Pole**  
*Bioinformatics and Modelling :  
from Genomes to Networks*